Dynamics of Complex Networks II: The Percolation problem

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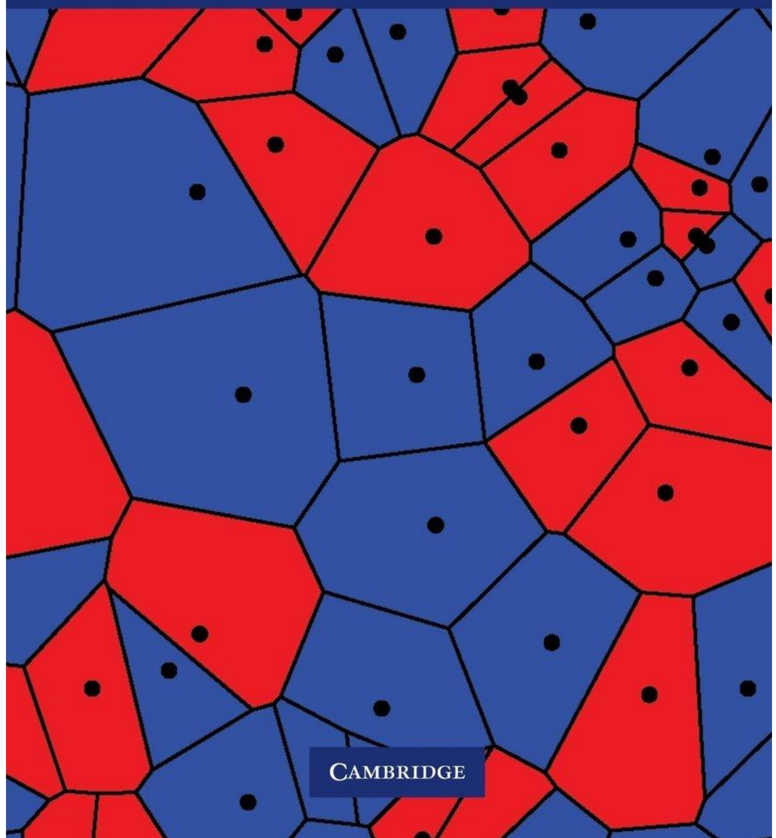
The percolation problem

PERCOLATION

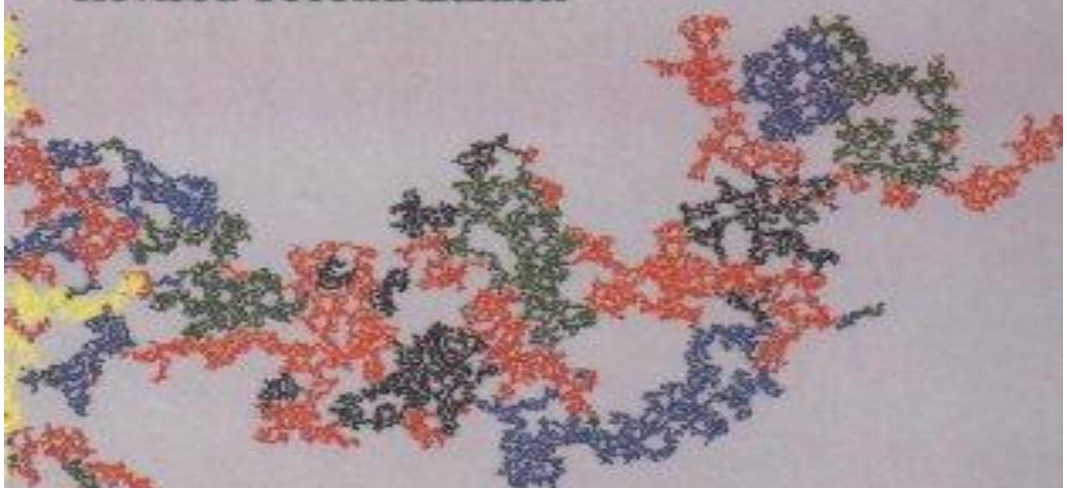
Béla Bollobás *and* Oliver Riordan



CAMBRIDGE

Introduction to PERCOLATION THEORY

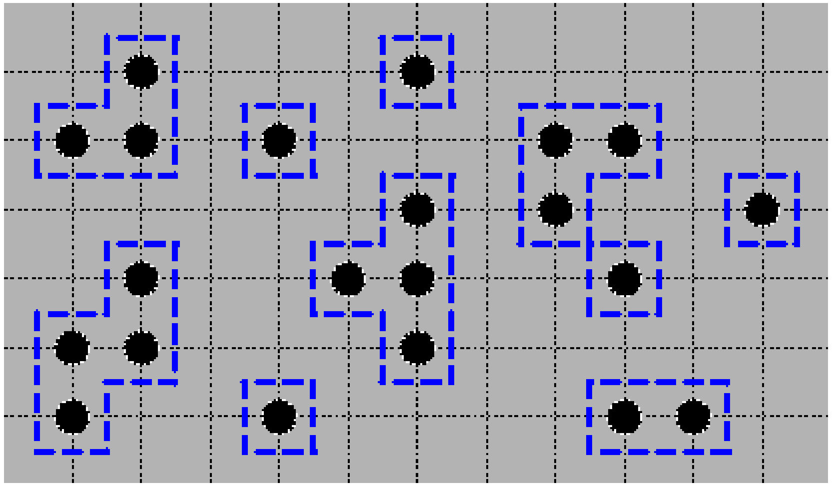
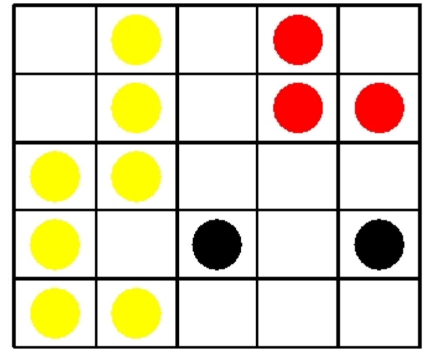
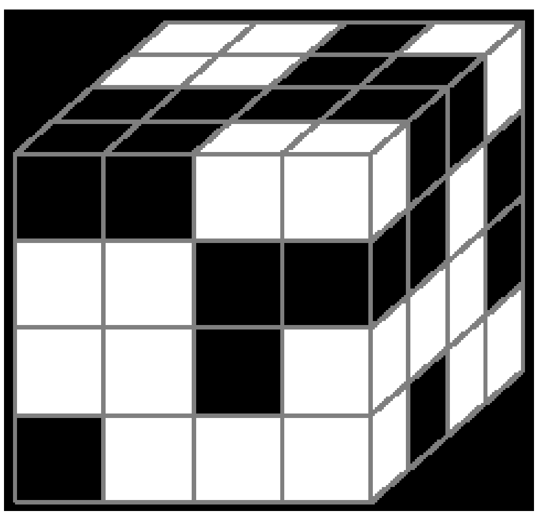
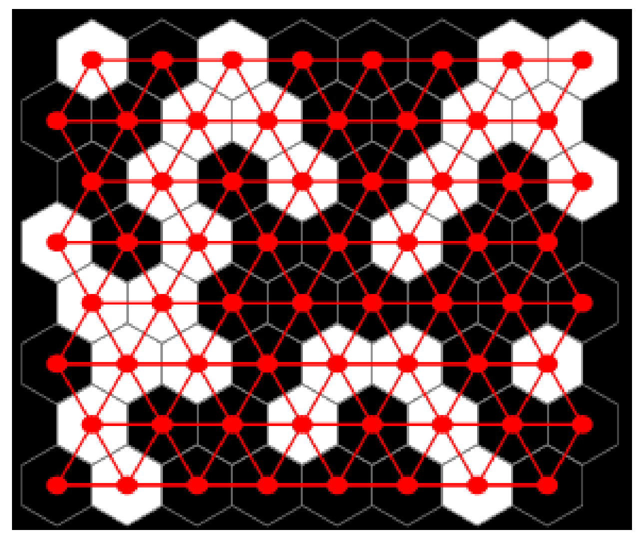
Revised Second Edition



DIETRICH STAUFFER AND
AMNON AHARONY

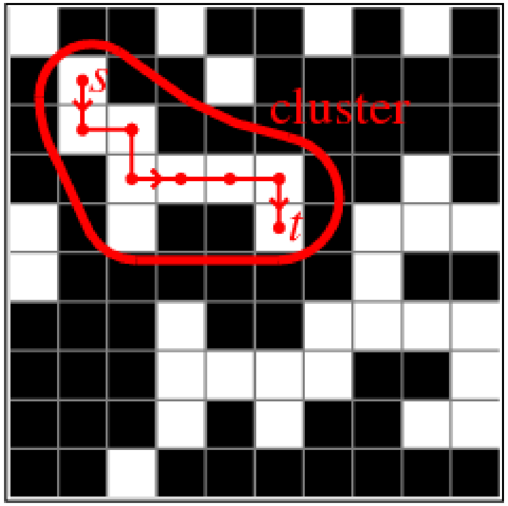
What is the problem?

- system made of 2 types of entities
- open/closed, true/false, conducting/insulating
- randomly mixed
- fixed ratio of open/closed, called "p"
- p in the range $0<p<1$
- adjacent entities of same type form clusters
- clusters depend on topology
- can be on lattice sites or on lattice bonds

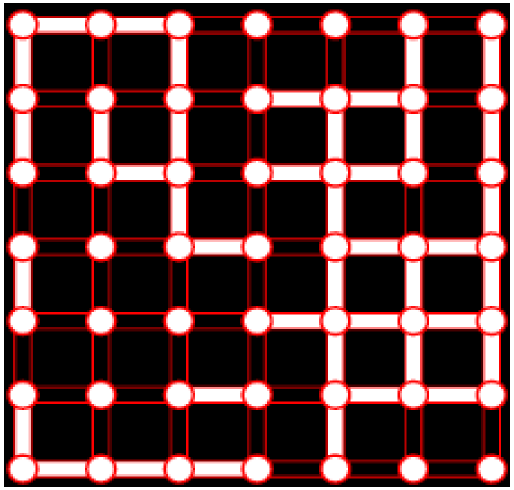





Site or bond percolation

site

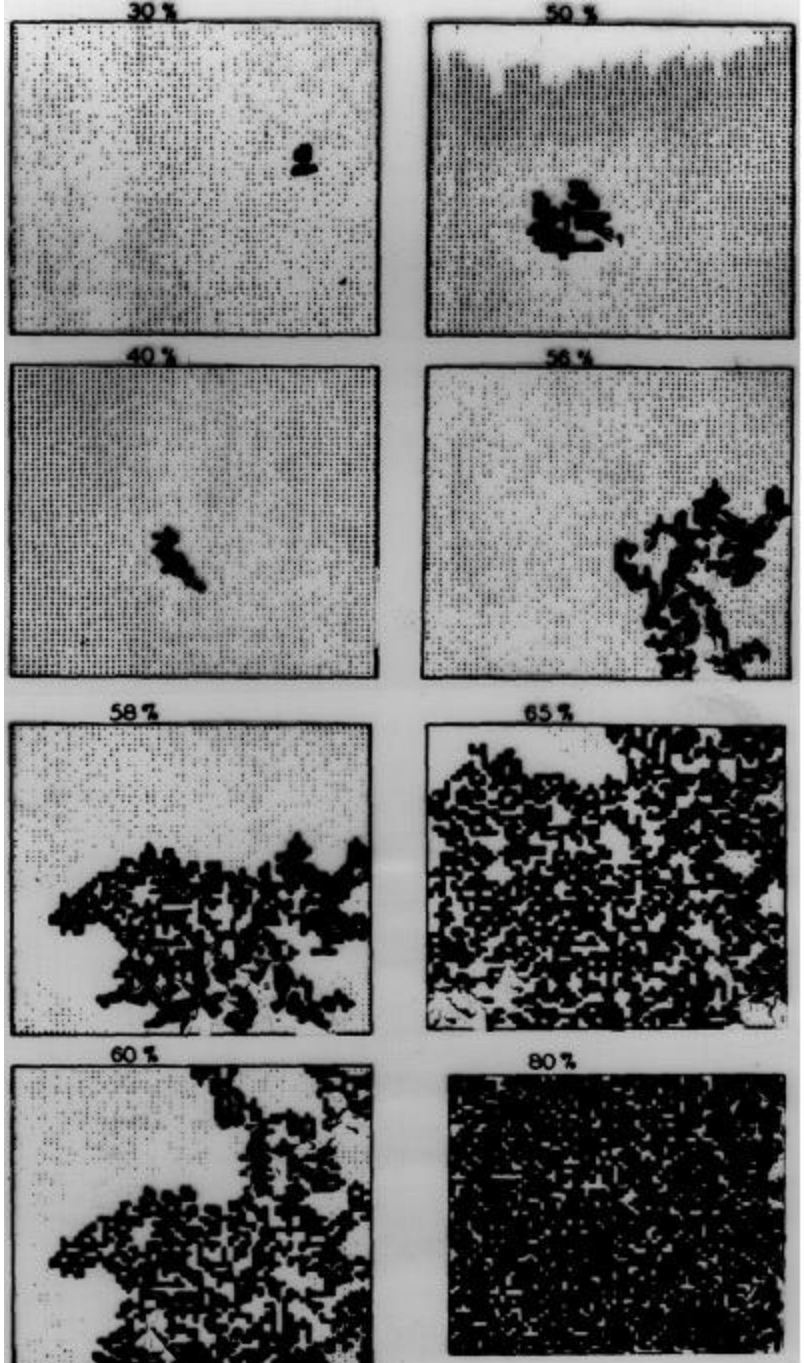


bond



Percolation phase transition

- focus on largest cluster only
- size increases abruptly at the critical point
- system goes through a phase transition from “insulating” to “conducting”
- 2^{nd} order phase transition, $\Delta H=0$


30 %
50 %
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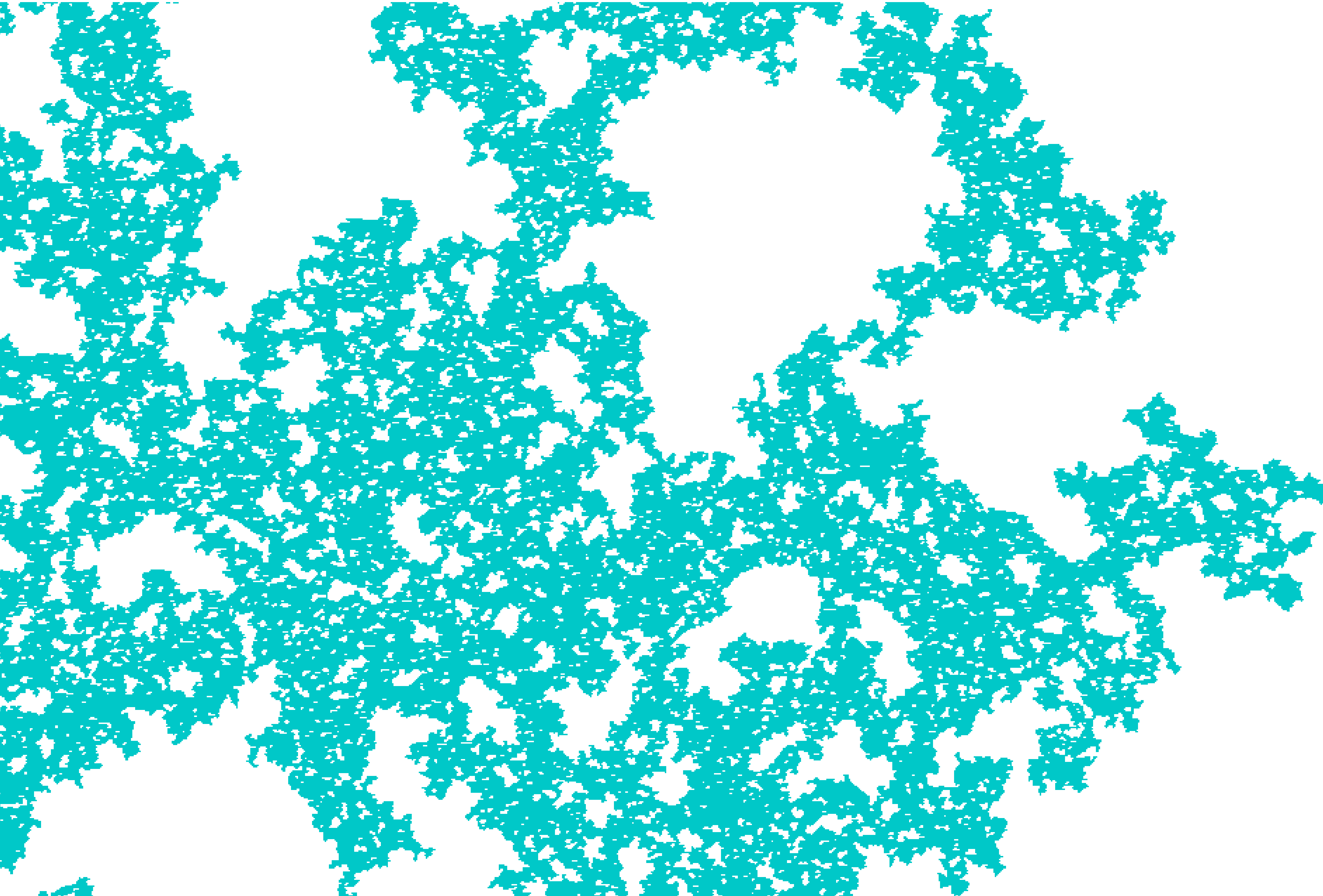


Percolation simulation

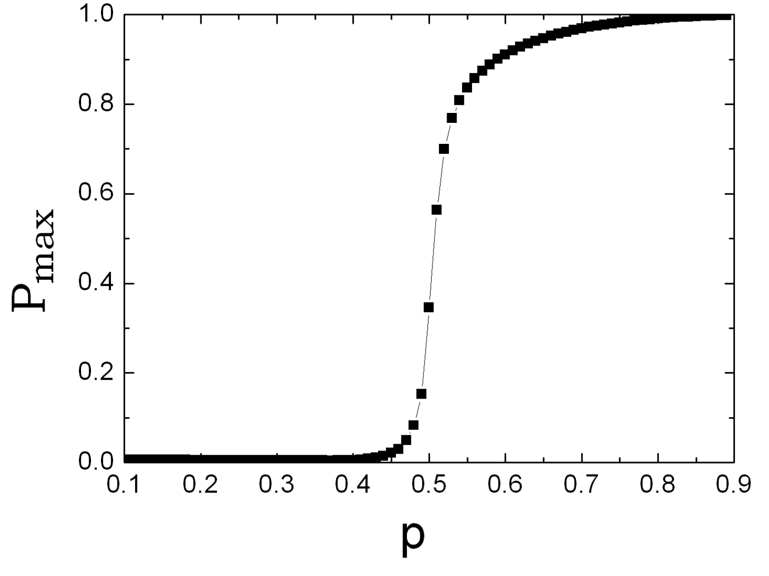
$$P_{\max} = \frac{m_{\max}}{pN^2}$$

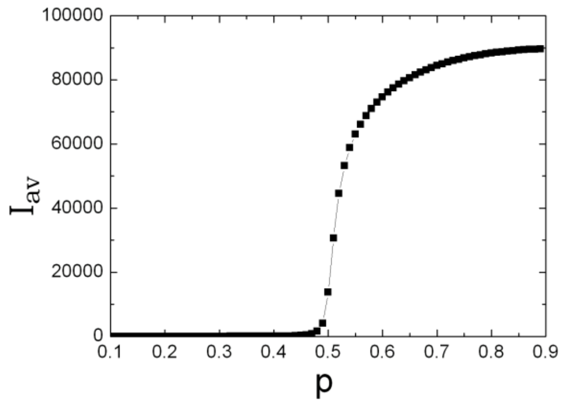
$$I_{\alpha v} = \sum_{m=1}^{m_{\max}} \frac{i_m m^2}{pN^2}$$

$$I'_{av} = I_{av} - \frac{i_{\max} * m_{\max}^2}{p * N^2}$$

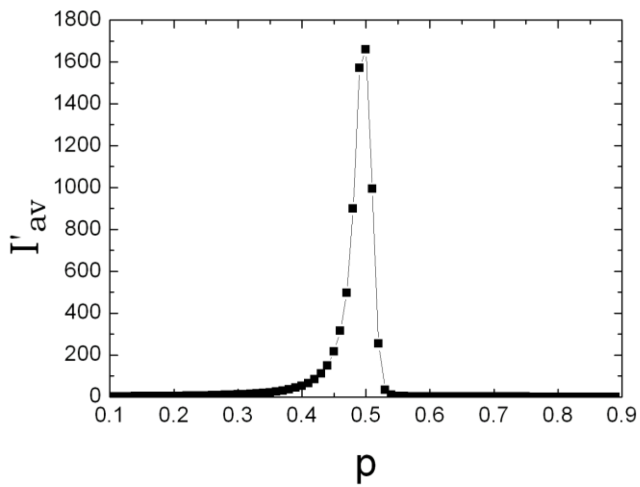
$$I'_{av} = \sum_{m=1}^{m-m_{max}} \left(\frac{i_{\mathrm{m}} \cdot m^2}{p \cdot N^2} \right)$$

P(max)





100000
80000
60000
40000
20000
0
I_{av}
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
p



1800
1600
1400
1200
1000
800
600
400
200
0
I'_{av}
0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9
p

How can we estimate p_c?

- several techniques have been developed
- square lattice (site percolation) p_c =0.5927……
- cannot be proven analytically
- square lattice (bond percolation) p_c =0.5000
- simple cubic(site) p_c =0.3116…
- simple cubic (bond) p_c =0.2488…
- p_c strongly depends on the lattice type
- the more nearest neighbors, the lower the p_c

Cluster Multiple Labeling Technique (CMLT)

- sweep the lattice from one end to the other
- for every cluster that appears give a different index number
- everytime 2 clusters join, they become one cluster
- “brute force” method: go back and merge the index numbers of the 2 clusters into 1 index number only. Need to sweep entire lattice
- CMLT method: need only a single sweep for the same job
- Invented by Hoshen (1976)

What happens when 2 clusters coalesce

- we need to add the 2 sizes into 1
- we change the label of the index, but NOT the index itself

Before the joining:

L(1)=1, L(2)=2, L(3)=3…..

____

After joining:

L(3)=2………

1	0	1	1	0	0	0	1	1	0	1	0	1	0	0	0	1	1	0	1
1	1	1	1	0	1	0	0	0	1	0	1	1	1	1	1	1	0	1	1
1	0	1	0	1	1	0	0	0	1	1	1	1	1	1	1	1	0	1	0
0	1	1	1	1	0	0	1	1	1	1	0	1	1	1	1	1	0	1	1
0	0	0	0	1	1	1	1	0	1	0	1	1	0	0	0	1	0	1	1
0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	1	1	1	0
1	1	1	0	0	1	1	1	1	0	1	1	0	1	1	1	0	0	1	0
1	1	1	0	1	1	1	1	1	0	1	0	1	0	0	0	0	0	1	1
1	1	0	0	1	1	0	0	0	0	1	1	0	1	0	0	0	0	0	1
1	0	1	0	0	1	0	1	1	1	0	0	0	1	1	0	0	0	1	1
1	1	0	1	1	0	1	1	1	1	0	1	0	0	1	1	1	0	0	0
0	1	1	1	1	0	1	0	1	0	1	0	1	0	1	1	0	0	0	1
1	1	1	0	0	1	1	0	0	0	0	0	1	0	0	1	0	1	0	1
1	1	1	1	1	0	0	1	1	0	1	1	1	1	1	0	1	0	0	1
0	1	0	1	1	0	1	1	1	1	1	1	1	1	1	0	0	1	0	1
0	1	0	1	1	1	1	0	1	1	0	1	0	1	0	1	0	1	1	1
1	0	1	1	0	0	0	1	1	1	1	0	0	0	1	1	0	1	1	1
1	1	1	0	0	0	1	0	1	0	1	0	0	1	0	0	1	1	0	1
0	1	1	1	0	0	1	0	0	1	0	0	0	1	0	1	1	0	1	0
0	0	1	0	1	0	0	1	1	0	0	0	1	1	1	0	0	1	1	1

Part (a)

1	0	6	5	0	0	0	18	18	0	26	0	33	0	0	0	43	31	0	52
1	1	1	5	0	13	0	0	0	24	0	30	11	31	31	31	31	0	49	31
1	0	1	0	10	5	0	0	0	24	11	11	11	31	31	31	31	0	49	0
0	5	5	5	5	0	0	19	11	11	11	0	11	31	31	31	31	0	49	31
0	0	0	0	5	5	11	11	0	11	0	31	31	0	0	0	31	0	49	31
0	0	0	0	0	5	11	11	0	0	0	0	0	0	0	0	31	31	31	0
2	2	3	0	0	5	11	11	11	0	27	27	0	37	37	37	0	0	31	0
2	2	3	0	11	11	11	11	11	0	27	0	34	0	0	0	0	0	31	31
2	2	0	0	11	11	0	0	0	0	27	27	0	38	0	0	0	0	0	31
2	0	7	0	0	11	0	20	14	14	0	0	0	38	38	0	0	0	50	50
2	2	0	9	4	0	15	14	14	14	0	32	0	0	38	38	38	0	0	0
0	2	3	3	4	0	15	0	14	0	28	0	35	0	38	38	0	0	0	53
3	3	3	0	0	14	14	0	0	0	0	0	35	0	0	38	0	46	0	53
3	3	3	3	4	0	0	21	4	0	29	22	22	22	22	0	44	0	0	53
0	3	0	3	4	0	16	4	4	22	22	22	22	22	22	0	0	47	0	53
0	3	0	3	4	4	4	0	4	22	0	22	0	22	0	41	0	47	42	42
4	0	8	4	0	0	0	22	22	22	22	0	0	0	40	40	0	47	42	42
4	4	4	0	0	0	17	0	22	0	22	0	0	39	0	0	45	42	0	42
0	4	4	4	0	0	17	0	0	25	0	0	0	39	0	42	42	0	51	0
0	0	4	0	12	0	0	23	23	0	0	0	36	36	36	0	0	48	48	48

Part (b)

50	0	50	50	0	0	0	18	18	0	26	0	50	0	0	0	50	50	0	50
50	50	50	50	0	50	0	0	0	50	0	50	50	50	50	50	50	0	50	50
50	0	50	0	50	50	0	0	0	50	50	50	50	50	50	50	50	0	50	0
0	50	50	50	50	0	0	50	50	50	50	0	50	50	50	50	50	0	50	50
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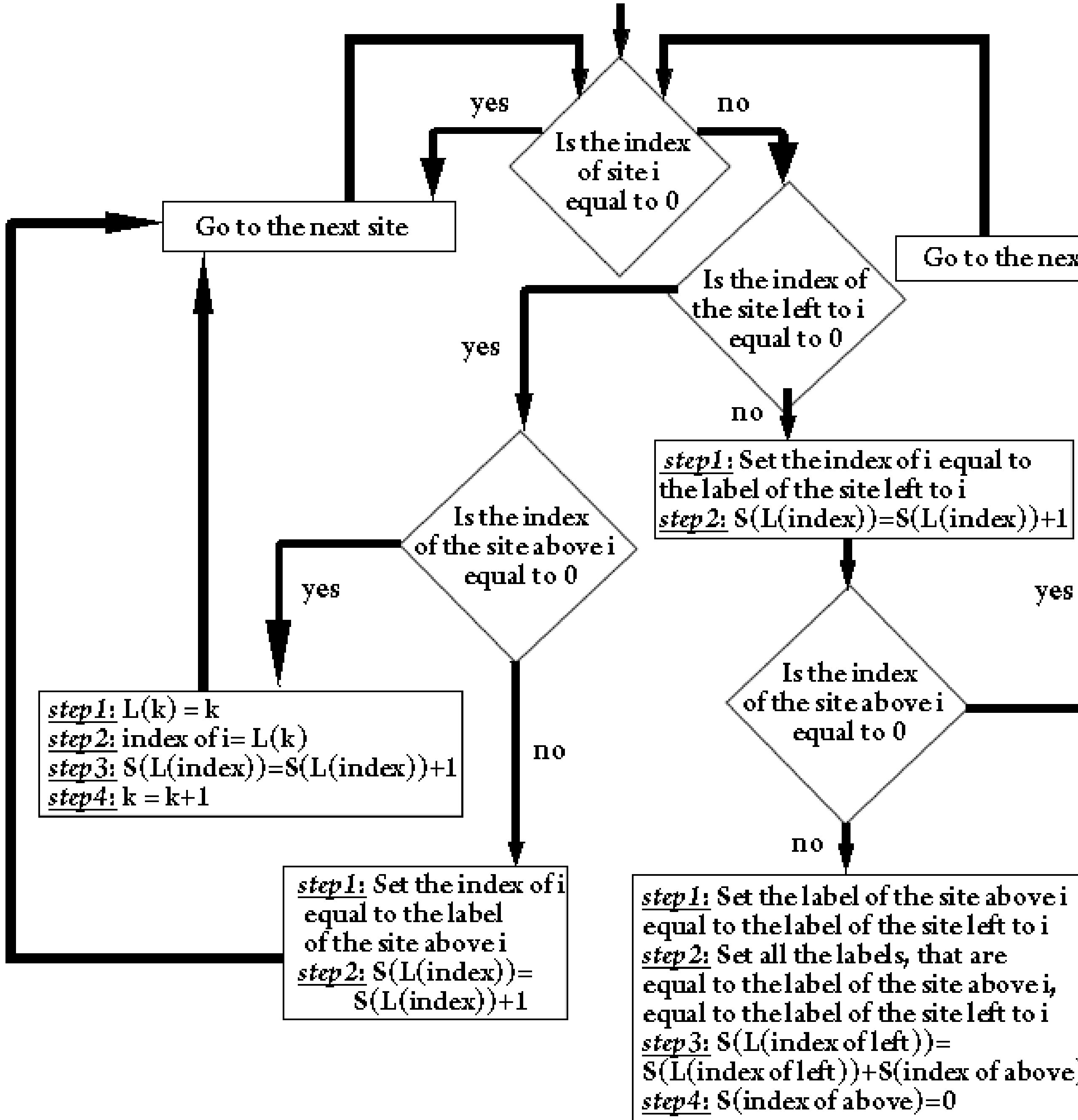
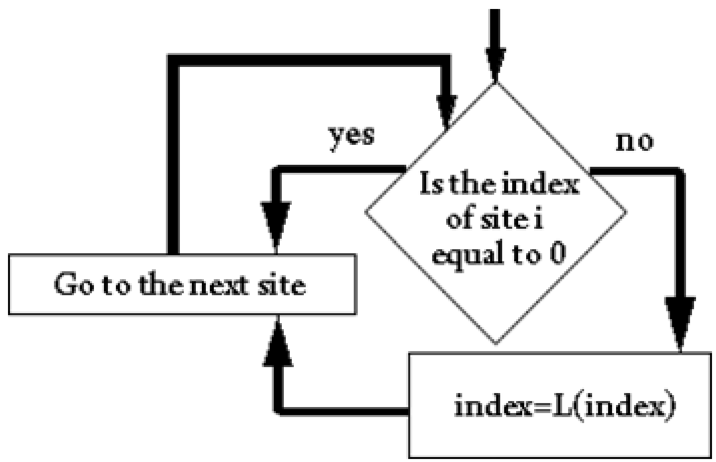
0	0	0	0	0	50	50	50	0	0	0	0	0	0	0	0	50	50	50	0
22	22	22	0	0	50	50	50	50	0	27	27	0	37	37	37	0	0	50	0
22	22	22	0	50	50	50	50	50	0	27	0	34	0	0	0	0	0	50	50
22	22	0	0	50	50	0	0	0	0	27	27	0	38	0	0	0	0	0	50
22	0	7	0	0	50	0	14	14	14	0	0	0	38	38	0	0	0	50	50
22	22	0	22	22	0	14	14	14	14	0	32	0	0	38	38	38	0	0	0
0	22	22	22	22	0	14	0	14	0	28	0	22	0	38	38	0	0	0	42
22	22	22	0	0	14	14	0	0	0	0	0	22	0	0	38	0	46	0	42
22	22	22	22	22	0	0	22	22	0	22	22	22	22	22	0	44	0	0	42
0	22	0	22	22	0	22	22	22	22	22	22	22	22	22	0	0	42	0	42
0	22	0	22	22	22	22	0	22	22	0	22	0	22	0	40	0	42	42	42
22	0	22	22	0	0	0	22	22	22	22	0	0	0	40	40	0	42	42	42
22	22	22	0	0	0	17	0	22	0	22	0	0	36	0	0	42	42	0	42
0	22	22	22	0	0	17	0	0	25	0	0	0	36	0	42	42	0	48	0
0	0	22	0	12	0	0	23	23	0	0	0	36	36	36	0	0	48	48	48

Part (c)



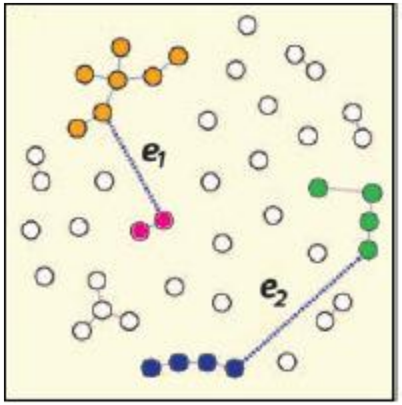
Is the index of site i equal to 0
yes
no
Go to the next site
Is the index of the site left to i equal to 0
Go to the next site
yes
no
step1: Set the index of i equal to the label of the site left to i
step2: S(L(index))=S(L(index))+1
Is the index of the site above i equal to 0
yes
yes
Is the index of the site above i equal to 0
step1: L(k) = k
step2: index of i= L(k)
step3: S(L(index))=S(L(index))+1
step4: k = k+1
no
no
step1: Set the index of i equal to the label of the site above i
step2: S(L(index))= S(L(index))+1
step1: Set the label of the site above i equal to the label of the site left to i
step2: Set all the labels, that are equal to the label of the site above i, equal to the label of the site left to i
step3: S(L(index of left))= S(L(index of left))+S(index of above)
step4: S(index of above)=0

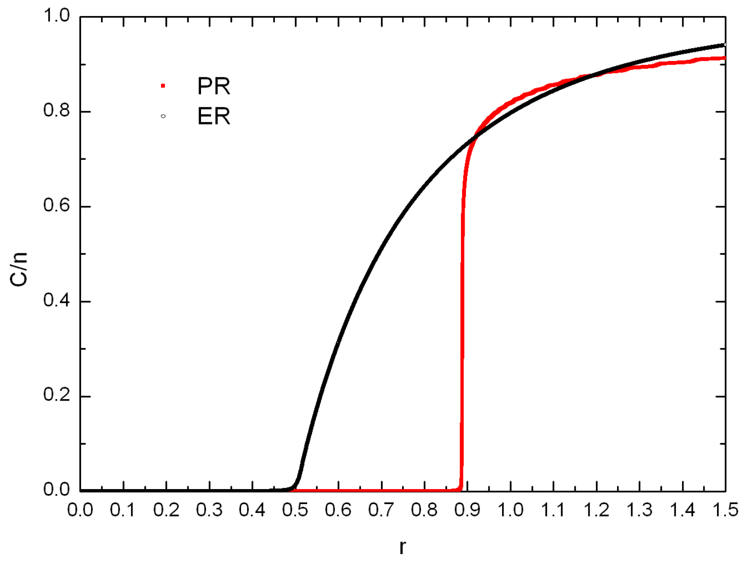

yes
no
Is the index
of site i
equal to 0
Go to the next site
index=L(index)

Achlioptas process

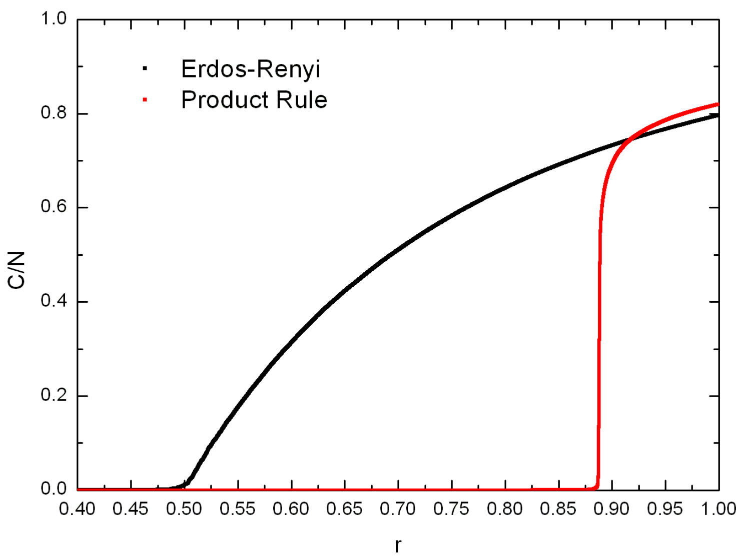
- developed in 2010
- new method of preparing the system
- use probe sites and fill lattice in such a way as to delay the criticality

Achlioptas process - product rule

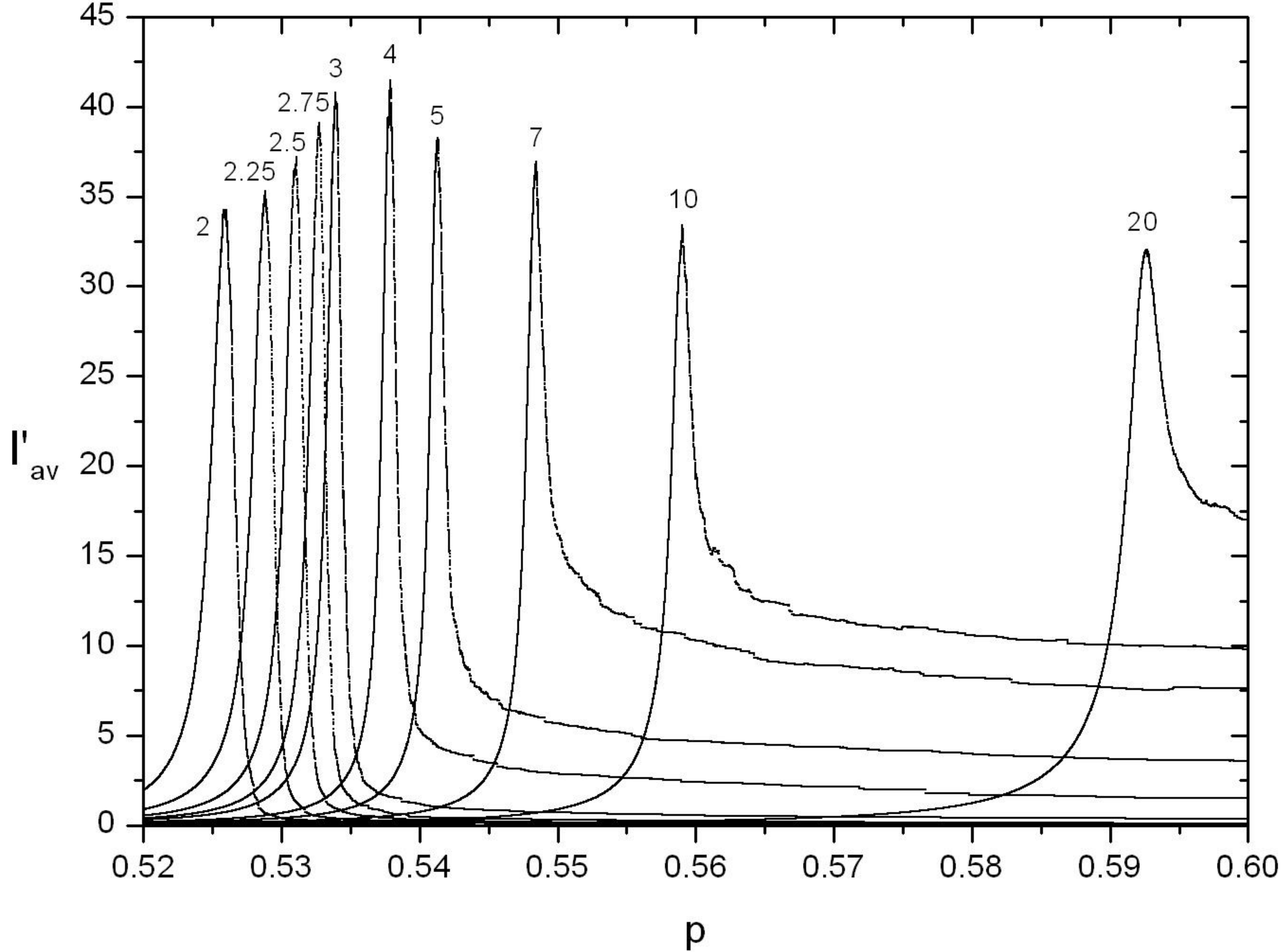




1.0
0.8
0.6
0.4
0.2
0.0
C/n
PR
ER
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0 1.1 1.2 1.3 1.4 1.5
r



1.0
0.8
0.6
0.4
0.2
0.0
C/N
Erdos-Renyi
Product Rule
0.40 0.45 0.50 0.55 0.60 0.65 0.70 0.75 0.80 0.85 0.90 0.95 1.00
r



45
40
35
30
25
20
15
10
5
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I'av
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0.58
0.59
0.60
p
2
2.25
2.5
2.75
3
4
5
7
10
20